Student details

Name: _____

Mark: _____

2024

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Extension 1

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 80

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 12

70 marks

- Attempt Questions 11 – 31
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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1 Which of the following is the solution for x: $\frac{x - 4}{3 - x} \geq 0$?

(A) $x \in (-3, 4]$

(B) $x \in (-\infty, 3] \cap [4, \infty)$

(C) $x \in (3, 4]$

(D) $x \in (-\infty, -3] \cap [4, \infty)$

2 Which of the following is the unit vector between the points (–5,–1) and (–2,3)?

(A) $-35\underset{\sim}{i} + 20\underset{\sim}{j}$

(B) $\frac{4}{5}\underset{\sim}{i} - \frac{7}{5}\underset{\sim}{j}$

(C) $-7\underset{\sim}{i} + 4\underset{\sim}{j}$

(D) $\frac{3}{5}\underset{\sim}{i} + \frac{4}{5}\underset{\sim}{j}$

3 Consider the equation $\sqrt{3}\sin x + \sqrt{2}\cos x = 2.$?

When using the auxiliary (transformation) method to solve for x, which one of the following would be an appropriate part of the working out?

(A) Let $\sqrt{3}\sin x + \sqrt{2}\cos x = R\sin(x+\alpha)$, where $0^\circ < \alpha < 90^\circ$.

(B) Let $\sqrt{3}\sin x + \sqrt{2}\cos x = R\cos(x-\alpha)$, where $0^\circ < \alpha < 90^\circ$.

(C) Either (A) or (B).

(D) Neither (A) or (B).

4 What is the area enclosed between curves $y = x$ and $y = \sqrt{x}$?

(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{6}$

5 An object is projected at an angle of θ° to the ground with initial velocity of u ms^{-1}. If the object is initially on the ground, and assuming gravity of g ms^{-2}, what is the object's maximum vertical height over its time of flight?

(A) $\frac{u\sin\theta}{g}$

(B) $\frac{2u\sin\theta}{g}$

(C) $\frac{u^2\sin^2\theta}{2g}$

(D) $\frac{u^2\sin 2\theta}{g}$

6 If $\cos\theta = \frac{3}{7}$ and $\sin\theta < 0$, what is the exact value of $\sin 2\theta$?

(A) $\frac{2\sqrt{10}}{3}$

(B) $-\frac{2\sqrt{10}}{7}$

(C) $-\frac{6\sqrt{10}}{49}$

(D) $-\frac{12\sqrt{10}}{49}$

7 Which of the following is the range of the function $f(x) = a + b\cos^{-1}(cx)$, where a, b and c are constants?

(A) $\left[-\frac{1}{c}, \frac{1}{c}\right]$

(B) $[0, a\pi]$

(C) $[0, b + c\pi]$

(D) $[a, a + b\pi]$

8 Which of the following equations may be appropriate to model uninhibited population growth?

(A) $P = 700e^{-0.06t}$

(B) $P = 2000 - 540e^{-0.85t}$

(C) $P = 1800e^{0.2t}$

(D) $P = \frac{9500}{1 + 4e^{-9t}}$

9 Seven different coloured jellybeans were to be placed into three separate jars. How many unique ways can this be done?

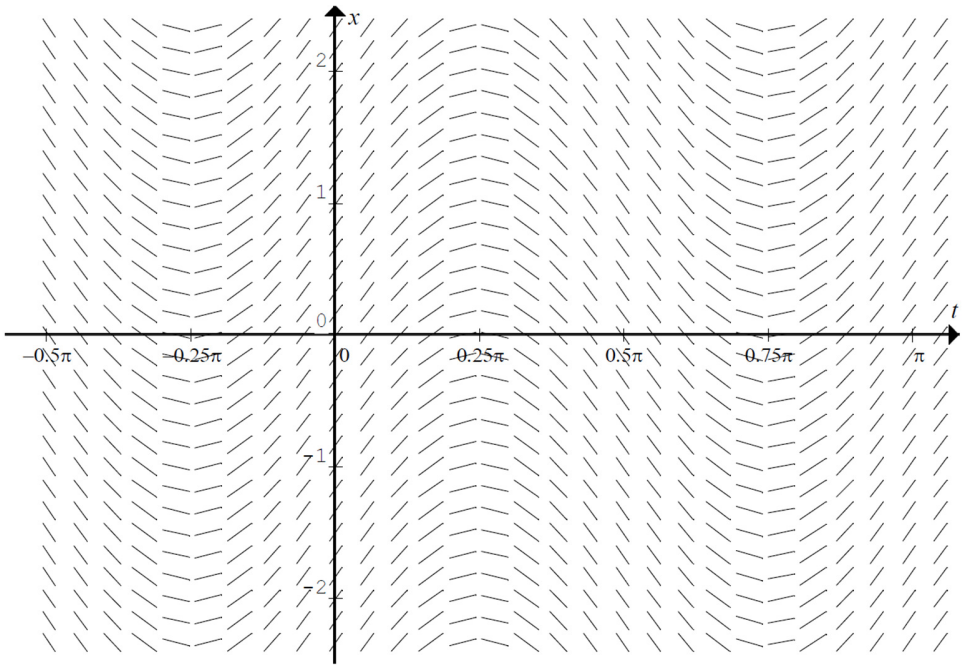
(A) 35

(B) 5,040

(C) 30,240

(D) 181,440

10 Which of the following could be the differential equation represented by the slope field shown?



(A) $\frac{dx}{dt} = \sin(2t)$

(B) $\frac{dx}{dt} = 2\cos(2t)$

(C) $\frac{dx}{dt} = -\frac{1}{2}\cos(t)$

(D) $\frac{dx}{dt} = \sin\left(\frac{t}{2}\right)$

Section II

60 marks
Attempt Questions 11–30

In Questions 11–30, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Differentiate with respect to x: $y = \tan^{-1}\left(x^3 + 2\right)$. **2**

Question 12

Consider the two vectors $\underset{\sim}{a} = \begin{pmatrix} -4 \\ -1 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 6 \\ -3 \end{pmatrix}$.

(a) Find $\left|\underset{\sim}{a}\right|$. **1**

(b) Find the size of the acute angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$, rounding your solution to the nearest degree. **1**

(c) Find $proj_{\underset{\sim}{a}}\underset{\sim}{b}$. **1**

Question 13

Find the exact value of $\sin\left(2\tan^{-1}\frac{3}{5}\right)$. **3**

Question 14

Find the coefficient of x^2 in the expansion of $\left(2x - \frac{3}{x^2}\right)^8$. **3**

Question 15

How many unique arrangements of the letters in the word **EXERCISE** are possible if:

(a) No restrictions applied? **1**

(b) The arrangement had all E's next to each other? **1**

(c) The arrangement began and ended with the letter E? **1**

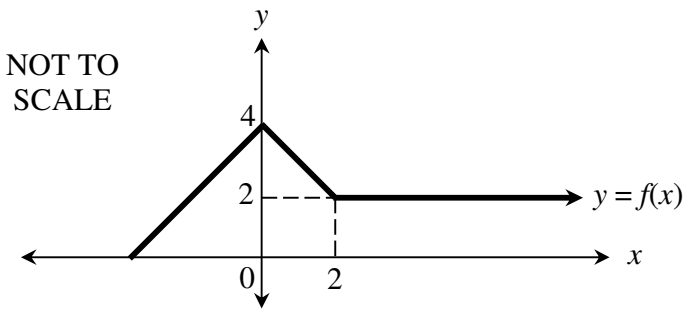
Question 16

Using the substitution $t = \tan\frac{\theta}{2}$ solve for θ, where $\pi \le \theta \le 2\pi$: $4\cos\theta - 3\sin\theta = 1$. **3**

Round your solution(s) to the nearest minute.

Question 17

The diagram shows the graph of a function $f(x)$.



Sketch the following curves on separate diagrams, showing all key features.

(a) $y = -f(x)$ **1**

(a) $y = 3f(x)$ **1**

(b) $y = \dfrac{1}{f(x)}$ **2**

(c) $y = \sqrt{f(|x|)}$ **3**

Question 18

Find the Cartesian equation of the following parametric equations: **2**

$$x = 3\cos\theta,\ y = 3\sin\theta,\ \text{where } \pi \le \theta \le 2\pi.$$

Question 19

Find the solution to the differential equation $\frac{dy}{dx} - \frac{\cos^2 y}{x^2 + 4} = 0$, given $y = \frac{\pi}{4}$ when $x = \frac{2}{\sqrt{3}}$. **3**

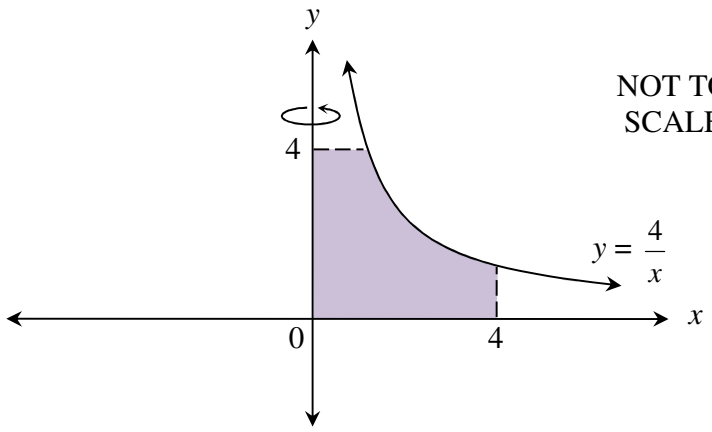
Express your solution in the form $y = f(x)$.

Question 20

Use the substitution $u = \sqrt{x+1}$ to evaluate $\int_3^8 \frac{x}{\sqrt{x+1}}\,dx$. **3**

Question 21

The area enclosed by the curve $y = \frac{4}{x}$, the x-axis, y-axis and the lines $x = 4$ and $y = 4$ is rotated about the y-axis, as shown in the diagram below. **3**



Find the volume formed by the rotation.

Question 22

Evaluate $\int_0^{\frac{\pi}{8}} \sin 5x \cos 3x \, dx$. **3**

Question 23

The surface area of a sphere is increasing at a rate of 4.2 mm^2/s. If the radius is currently 1.6mm, find the rate at which the sphere's volume is increasing at. Round your solution to one decimal place. **3**

Question 24

The polynomial $P(x) = x^3 + 12x^2 + Ax + B$, where A and B are real numbers, has roots α, $(\alpha + 2)$ and $(\alpha - 2)$.

(a) Find the value of α. **1**

(b) Hence, or otherwise, find the value of A and B. **2**

Question 25

Five letters from the word "DEVELOPER" were chosen at random to form a new five-letter arrangement. **3**

How many unique arrangements are possible?

Question 26

Prove by mathematical induction that $2^n + 5^n + 7^n$ is divisible by 7 if n is an <u>odd integer</u>. **3**

Question 27

Splinta wants to start a turtle farm with the four turtles he found by the lake. After renting out a small piece of land and buying the required resources, the number of turtles N in his farm grew over t years according to the differential equation:

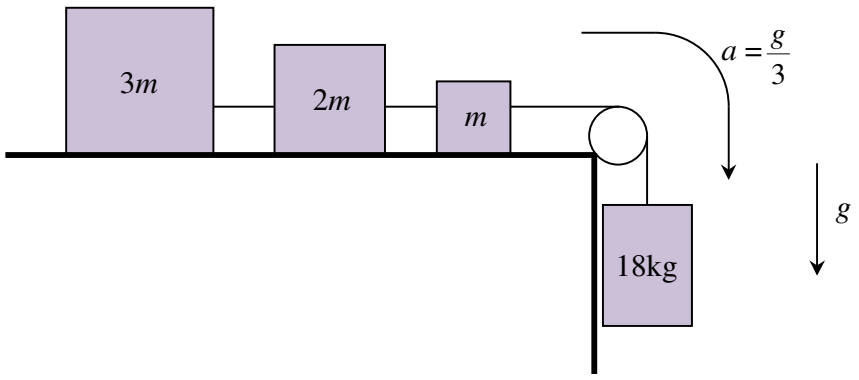
$$\frac{dN}{dt} = \frac{(N+5)(N+2)}{N-1}.$$

(a) Show that: $\dfrac{N-1}{(N+5)(N+2)} = \dfrac{2}{N+5} - \dfrac{1}{N+2}$. **1**

(b) How long will it take for Splinta to have over 50,000 turtles? Round your solution to the nearest year. **3**

Question 28

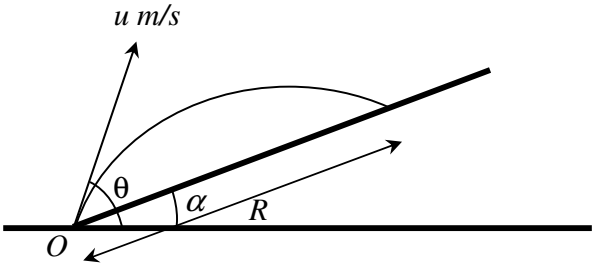
Three objects with mass of $3m$ kg, $2m$ kg and m kg were connected to another object of 18kg hanging off the edge of the table by light inextensible strings in a pulley system. The objects were accelerating at a rate of $\frac{g}{3}$ m/s^2 along a smooth table surface, where g m/s^2 represents gravity, as shown in the diagram. **3**



By resolving forces, find the value of m.

Question 29

A projectile was launched from a point O on the ground with initial velocity of u m/s and an angle of θ to the horizontal. The projectile was launched onto a plane inclined at α to the horizontal travelling a distance of R metres on the plane, as shown in the diagram below.



After t seconds, the horizontal (x) and vertical (y) displacements of the projectile is given as follows (**DO NOT PROVE THESE**):

$$x = u\cos\theta t \qquad \text{and} \qquad y = -\frac{gt^2}{2} + u\sin\theta t$$

where gravity is g m/s^2.

(a) Show that the time taken for the projectile to land on the inclined plane T is: **1**

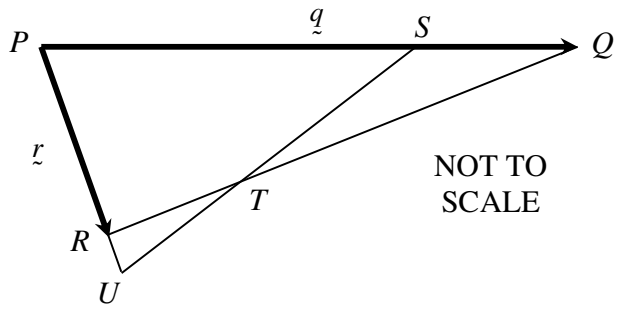
$$T = \frac{R\cos\alpha}{u\cos\theta}.$$

(b) Show that the Cartesian equation of the projectile is: $y = -\dfrac{gx^2}{2u^2\cos^2\theta} + \dfrac{x\sin\theta}{\cos\theta}$. **1**

(c) Hence, or otherwise, show that: $R = \dfrac{2u^2\cos\theta\sin(\theta-\alpha)}{g\cos^2\alpha}$. **2**

Question 30

In ΔPQR, S is a point on PQ such that $PS : SQ = 2 : 1$ and T is a point on QR such that $QT : TR = 2 : 1$. When ST is extended it meets the extension of PR at the point U, as shown in the diagram below.



Let $\overrightarrow{PQ} = \underset{\sim}{q}$ and $\overrightarrow{PR} = \underset{\sim}{r}$.

(a) Show that $\overrightarrow{ST} = \frac{2}{3}\underset{\sim}{r} - \frac{1}{3}\underset{\sim}{q}$. **2**

(b) By letting $\overrightarrow{PU} = \lambda\overrightarrow{PR}$ and $\overrightarrow{SU} = \mu\overrightarrow{ST}$, show that $PR : RU = 3 : 1$. **3**

Question 31

Consider the functions $f(x) = \sqrt{16 - x^2}$ and $g(x) = x^2 - 5$.

(a) Find the expression for $f\left(g(x)\right)$. **1**

(b) Hence, or otherwise, find the domain and range of $f\left(g(x)\right)$, expressing your solution in set notation. **3**

End of paper.