Student details

Name: ______________________

Mark: ______________________

2024

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

Mathematics Advanced

General Instructions

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Reference sheet is provided separately.
- Marks may be lost for poor working out and/or poor logic.

Total marks – 100

Section I Pages 2 – 5

10 marks

- Attempt Questions 1 – 10
- Circle the BEST solution.

Section II Pages 6 – 15

90 marks

- Attempt Questions 11 – 34
- Your responses should include relevant mathematical reasoning and/or calculations.

Section I

10 marks
Attempt Questions 1 – 10

Circle the BEST solution below for Questions 1 – 10.

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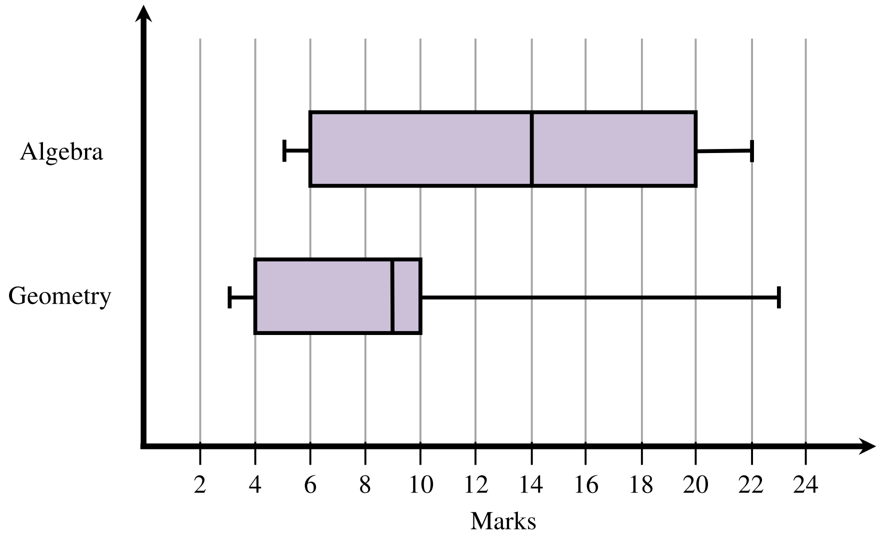
1 The following box plots summarises two class tests:



Which of the following is **FALSE**?

(A) The interquartile range in Algebra is higher than Geometry.

(B) The range in Algebra is higher than Geometry.

(C) The median in Algebra is higher than Geometry.

(D) There is an outlier mark in the Geometry.

2 What is the solution to $|x + 7| < 15$?

(A) $x \in (-22, 8)$

(B) $x \in [-22, 8]$

(C) $x \in (-\infty, -22] \cup [8, \infty)$

(D) $x \in (-\infty, -22) \cup (8, \infty)$

3 Consider a random variable X where E(X) = 9 and Var(X) = 4. Which of the following represents the values for $\mathrm{E}(3X + 7)$ and $\mathrm{Var}(3X + 7)$ respectively?

(A) 34 and 36.

(B) 34 and 16.

(C) 3 and 36.

(D) 3 and 16.

4 The function $f(x)$ is defined where the curve $y = 4x^2 - 24x + 31$ is monotonically decreasing. Which of the following represents the inverse function $y = f^{-1}(x)$?

(A) $x = 4y^2 - 24y + 31$

(B) $y = -\sqrt{\frac{1}{4}(x+5)} + 3$

(C) $y = \sqrt{\frac{x+5}{2}} - 3$

(D) $y = 4(x - 3)^2 - 5$

5 What is the domain for the function $f(x) = \dfrac{6}{\sqrt{x+4}}$?

(A) $x \in (-\infty, -4) \cup (-4, \infty)$

(B) $x \in (-\infty, -4)$

(C) $x \in (-4, \infty)$

(D) $x \in (-\infty, -24)$

6　Which of the following equates to approximating $\int_3^5 \log_{10} x \, dx$ using the Trapezoidal rule with five function values?

(A)　$\frac{1}{2}\log_{10}60$

(B)　$\frac{1}{4}\log_{10}147$

(C)　$\frac{1}{2}\log_{10}945$

(D)　$\frac{1}{4}\log_{10}59535$

7　In an exam, the average was 62.3% and the standard deviation was 11.8%. Assuming the marks were Normally distributed, what percentage of scores were above 38.7%?

(A)　95%

(B)　97.5%

(C)　99.7%

(D)　99.85%

8　A particle is moving along the *x*-axis in a straight line. After *t* seconds, the particle's velocity and acceleration is -5 ms^{-1} and -5 ms^{-2} respectively.

Which statement best describes the motion of the particle after *t* seconds?

(A)　The particle is moving to the left and is slowing down.

(B)　The particle is moving to the right and is slowing down.

(C)　The particle is moving to the left and is speeding up.

(D)　The particle is moving to the right and is speeding up.

9 Which of the following represents the total accumulated value of investing \$$P$ per year at the start of each year for n years into a superannuation fund that earns r % p.a.?

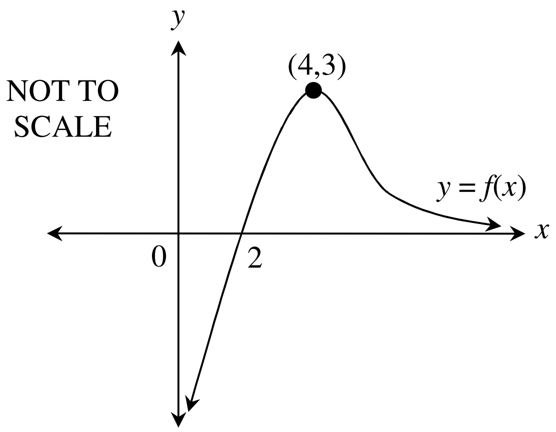
(A) $P\left[\dfrac{(1+r)^n - 1}{r}\right]$

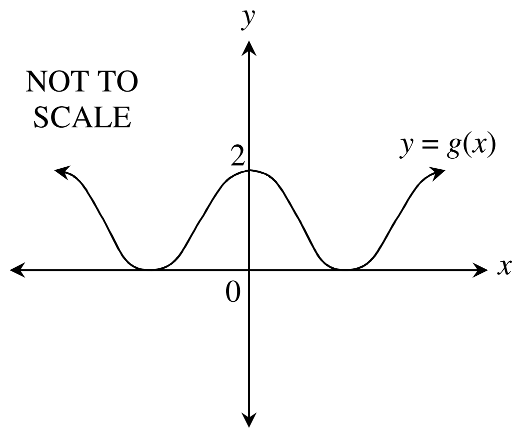
(B) $P(1+r)\left[\dfrac{(1+r)^n - 1}{r}\right]$

(C) $P\left[\dfrac{(1+r)^n - 1}{r(1+r)^n}\right]$

(D) $\dfrac{P}{(1+r)}\left[\dfrac{(1+r)^n - 1}{r(1+r)^n}\right]$

10 Consider the following functions:





Which of the following is the range of $f\left[g\left(x\right)\right]$?

(A) $[0,2]$

(B) $(-\infty,0]$

(C) $[2,3]$

(D) $(-\infty,4]$

Section II

90 marks
Attempt Questions 11–32

In Questions 11–32, your responses should include relevant mathematical reasoning and/or calculations.

Question 11

Solve for x: $(x+7)(3-x)=0$. **1**

Question 12

Differentiate the following with respect to x:

(a) $y=\sqrt[5]{x}$. **1**

(b) $y=\left(e^{5x}+4\right)^{7}$. **1**

(c) $y=\sqrt{x}\sin 2x$. **2**

(d) $y=\log_e\left(\tan 4x\right)$. **2**

Question 13

Find the limiting sum of in the series $24+12+6+\ldots$ **2**

Question 14

Draw a neat sketch of the curve $y=3\cos\frac{x}{2}$ for $x\in[0,2\pi]$, labelling all key features. **2**

Question 15

The probability distribution of a discrete random variable X is summarised in the following table:

x	2	5	7
$P(X = x)$	$2m + n$	0.3	$m + n$

where m and n are constants. The expected value of X is 4.4.

(a) Find the value of m and n. **2**

(b) Find the variance of X. **1**

Question 16

Find:

(a) $\int \sqrt{6x + 1}\, dx$. **1**

(b) $\int_{2}^{8} \frac{3}{x}\, dx$. **2**

(c) $\int \frac{16\cos 2x - 12\sin 3x}{4\sin 2x + 2\cos 3x}\, dx$. **2**

Question 17

Solve for x: $x^2 + 4x - \frac{6}{x^2 + 4x} = -5$. **3**

Question 18

The terms $(a - 2)$, a and $(a + 4)$ are consecutive terms in a geometric progression. **2**

Find the value of a.

Question 19

Find the equation of the tangent to the curve $y = \sin^3 x + 2$ at $x = \frac{\pi}{3}$. **3**

Question 20

Consider the series 185 + 179 + 173 + …

(a) Find the 9th term. **1**

(b) Show that the sum of the first n^{th} terms S_n is given by the equation $S_n = n(188 - 3n)$. **1**

(c) What the maximum number of terms such that the sum remains positive? **1**

Question 21

The population of echidnas in *Long Island* was protected by the local government in an attempt to help the population grow. At the start of 1998, it was estimated that there were 120 echidnas on the island. Scientists estimated the population (P) growth over t years to follow the following differential equation:

$$\frac{dP}{dt} = kP$$

where k is a constant.

(a) Verify that the equation $P = 120e^{kt}$ is a solution to the differential equation. **1**

(b) At the start of 2005, the population of echidnas grew to 850. Find the value of k, rounding your solution to three significant figures. **2**

Using your solution in (b),

(c) What was the rate of increase in the population of echidnas at the start of 2005? **1**

(d) Estimate the echidna on the island at the start of 2028. **1**

Question 22

The following table shows the values that follow a Standard Normal distribution.

z	First Decimal Place									
	.0	**.1**	**.2**	**.3**	**.4**	**.5**	**.6**	**.7**	**.8**	**.9**
0	0.5000	0.5398	0.5793	0.6179	0.6554	0.6915	0.7257	0.7580	0.7881	0.8159
1	0.8413	0.8643	0.8849	0.9032	0.9192	0.9332	0.9452	0.9554	0.9641	0.9713
2	0.9772	0.9821	0.9861	0.9893	0.9918	0.9938	0.9953	0.9965	0.9974	0.9981
3	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000

If Z is a random variable that is Standard Normally distributed, find the value of the following:

(a) $P(Z > 0.8)$. **1**

(b) $P(-2.3 \leq Z \leq 1.7)$. **2**

Question 23

A particle moves along a straight line where its velocity v ms^{-1} after time t seconds is given by the formula:

$$v = te^{t^2} - 8t.$$

Initially, the particle is 1 metre to the right of the origin O.

(a) In terms of t, find an expression for the particle's displacement x in metres. **2**

(b) Find the time(s) when the particle is at rest. **2**

(c) Find the particle's minimum displacement from the origin O, leaving your solution in exact form. **1**

Question 24

Solve for x, $x \in [0, 2\pi]$: $2\sin x + 1 = \operatorname{cosec} x$. **3**

Question 25

Consider two events, X and Y, where $P(X) = 0.65$, $P(Y) = 0.72$ and $P(X \cup Y) = 0.89$.

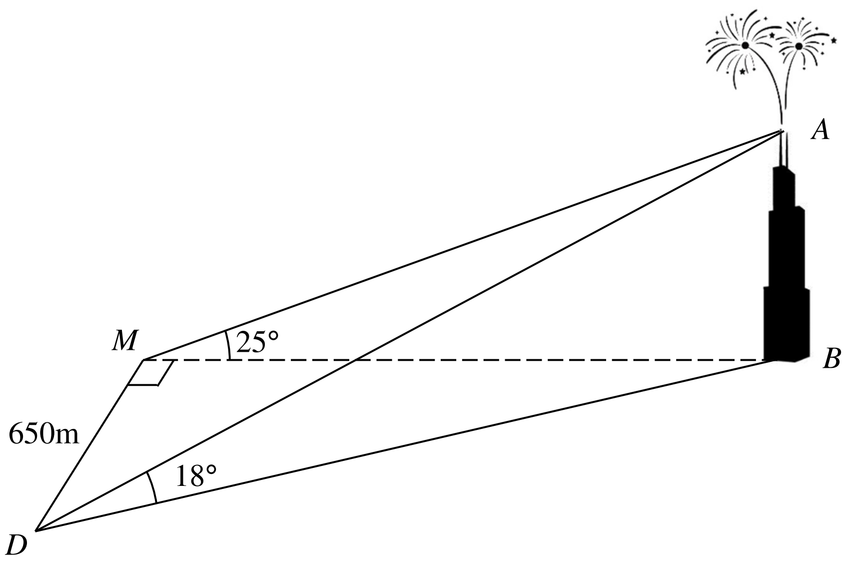
Find the value of:

(a) $P(X \cap Y)$. **1**

(b) $P(Y \mid X)$. **2**

Question 26

Yokal was a farmer who had planned a trip to the city *Shidoney* to see the fireworks on New Year's Eve. It was a known fact that the fireworks display originating from *Shidoney Tower* (*AB*) was something to behold. Yokal had read in a tourist brochure that two locations were the best vantage points to observe the fireworks – *Midnight Square* (*M*), which was due west of *Shidoney Tower*, and *Dusk Arena* (*D*), which was 650m due south of *Midnight Square*. The angles of elevation to the top of *Shidoney Tower* from *Midnight Square* and *Dusk Arena* are 25° and 18° respectively, as shown in the diagram below.



Let h be the height of *Shidoney Tower*.

(a) Show that $h = \dfrac{650}{\sqrt{\cot^2 18 - \cot^2 25}}$. **2**

(b) Hence, find the height of *Shidoney Tower*, rounding to the nearest metre. **1**

Question 27

Boar Dovstudis, minister of sport and recreation, received a worrying report regarding the amount of time students spent on studies versus time spent on pursuing sporting achievements. In particular, the report showed that as students get older, the focus on academic pursuits was heavily prioritised over participating in sporting activities. Boar decided to conduct a quick survey to understand the number of hours students participated in sporting activities (H) for a week across a range of student ages (A). The survey results were shown below:

Age of student (A)	No. of sporting hours (H)
17	3
15	10
14	18
15	22
12	17
16	1
18	4
16	13
12	17
15	3
14	20
13	16

By using the table above, and rounding your solution to two decimal places where required:

(a) State the dependent variable. **1**

(b) Determine the value of Pearson's correlation coefficient (r). **1**

(c) Using (b), describe the strength of the correlation between A and H. **1**

(d) Find the 'line of best fit', stating your solution as $H = \square + \square \times A$. **2**

(e) Using your solution to part (d), estimate the number of sporting hours for a 15 year old student, rounding your solution to the nearest hour. **1**

Question 28

The function $f(x) = (x+4)(x-3)$ underwent three transformations, as shown below: **3**

$$
\begin{aligned}
f(x) &= (x+4)(x-3) \\
&\rightarrow -(x+4)(x-3) \\
&\rightarrow -\left(\frac{x}{2}+4\right)\left(\frac{x}{2}-3\right) \\
&\rightarrow -\left(\frac{x}{2}+2\right)\left(\frac{x}{2}-5\right)
\end{aligned}
$$

Describe the three transformations made to $f(x)$.

Question 29

On his 20th birthday Tobi decides to start saving for his future. He decides to deposit \$2000 at the start of every year into a savings account that earns 4.5% p.a. On his 30th birthday, he decides to increase the amount to \$8000 every year, and on his 45th birthday, he increases it further to \$24,000 every year. **3**

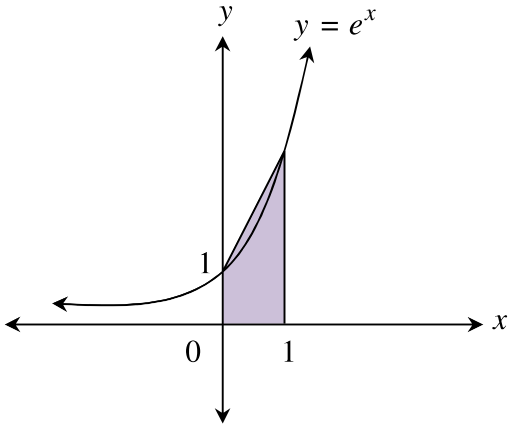
Below is a table that shows the future value of a \$1 annuity across different years and interest rates (p.a.).

Future Value of a \$1 Annuity							
End of Year	**Interest Rate (p.a.)**						
	3.0%	**3.5%**	**4.0%**	**4.5%**	**5.0%**	**5.5%**	**6.0%**
5	5.3091	5.3625	5.4163	5.4707	5.5256	5.5811	5.6371
10	11.4639	11.7314	12.0061	12.2882	12.5779	12.8754	13.1808
15	18.5989	19.2957	20.0236	20.7841	21.5786	22.4087	23.2760
20	26.8704	28.2797	29.7781	31.3714	33.0660	34.8683	36.7856
25	36.4593	38.9499	41.6459	44.5652	47.7271	51.1526	54.8645
30	47.5754	51.6227	56.0849	61.0071	66.4388	72.4355	79.0582
35	60.4621	66.6740	73.6522	81.4966	90.3203	100.2514	111.4348
40	75.4013	84.5503	95.0255	107.0303	120.7998	136.6056	154.7620
45	92.7199	105.7817	121.0294	138.8500	159.7002	184.1192	212.7435
50	112.7969	130.9979	152.6671	178.5030	209.3480	246.2175	290.3359

Using the table of values, find the total amount Tobi's savings account on his 50th birthday, rounding your solution to the nearest cent.

Question 30

The curve $y = e^x$ is shown in the diagram below. **3**



By considering the area of the shaded trapezium, show that $e < 3$.

Question 31

Kog was an engineer working for one of the world's most covert spy network. His latest project involved the invention of tiny, microscopic drones powered by friction, of which Kog had experimented with a vast number of different designs. Kog monitored the distance (*x*) in kms travelled by each design, and noted that the distribution broadly followed the following equation:

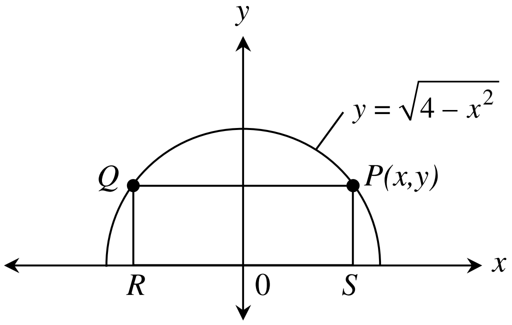
$$f(x) = \begin{cases} x^2\left(x^3 + 1\right)^2 & 0 \le x \le 1 \\ 152 - 144x & 1 < x \le 1\frac{1}{18} \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the function is a probability density function. **2**

(b) Find the distance represented by the 90^{th} percentile, rounding to three decimal place. **2**

Question 32

A rectangle $PQRS$ is inscribed inside the semi-circle $y = \sqrt{4 - x^2}$, where P and Q lie on the curve, as shown in the diagram below.



(a) If P has the coordinates (x,y), show that the area (A) of the rectangle is given by: **1**

$$A = 2x\sqrt{4 - x^2}.$$

(b) By differentiating the expression in (a), find the value of x that maximises the area of $PQRS$. Express your solution in exact form. **3**

Question 33

Hiro borrowed \$900,000 from a bank to finance his dream house. The bank offered Hiro a reducible interest rate of 3% p.a. where the loan will be repaid in equal monthly instalments of \$$M$ over a 25-year period. The amount owning on the loan after n months is denoted by A_n.

(a) Show that the amount owing after the n^{th} repayment is made is: **3**

$$A_n = 900000 \times 1.0025^n - 400M\left(1.0025^n - 1\right).$$

(b) Hence, or otherwise, show that the monthly repayment M is \$4267.90. **1**

(c) Since borrowing the loan from the bank, Hiro had made a number of smart decisions and has worked hard to be in a great financial position. After five years of monthly repayments, Hiro was in a position to repay more of the loan and decides that he wants to fully repay the loan in another 10 years (i.e. 15 years from the start of the loan). **3**

Find the <u>extra amount per month</u> on top of the amount in (b) that he would have to pay to achieve this, rounding your solution to the nearest cent.

Question 34

Consider the curve $y = \dfrac{x^3 + x^2 + x - 3}{x+1}$.

(a) Find the coordinates of the stationary points and determine their nature. **3**

(b) Show that $x^3 + x^2 + x - 3 = \left(x^2 + 1\right)\left(x+1\right) - 4$. **1**

(c) Find the equations of the vertical and oblique asymptotes. **2**

(d) Sketch the function $y = \dfrac{x^3 + x^2 + x - 3}{x+1}$, labelling all key features. **2**

End of paper.